The wedge form of relativistic dynamics

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It is commonly accepted that in hadronic or nuclear collisions at extremely high energies the shortest scales are explored. At the classical level, this property of the interaction is closely related to the Lorentz contraction of the fields of colliding particles which provides instantaneous switching the interaction on. I argue that the underlying quantum dynamics should be confined to within the light wedge of the two-dimensional plane where the first interaction takes place and suggest to include this property as the boundary condition for the quantum field theory which describes the collision process. Connection between the type of inclusive process and the temporal order of its dynamical evolution is discussed. The one-particle states and propagators of the perturbation theory for the scalar and fermion fields are found.

I. INTRODUCTION

In the previous paper [1], I suggested to view hadronic and nuclear collisions at extremely high energies in a new way; a strongly localized interaction performs a kind of spectral analysis of the initial bound states in terms of free quarks and gluons of the final one. Following this idea I re-formulated theory of the deep inelastic electron proton scattering (DIS) in the language of the quantum field kinetic theory [2] which escapes an intermediate parton phenomenology obviously used in calculation of the hadronic processes at high energies. The general scheme to calculate the quark and gluon distributions after the first hard collisions of heavy ions was traced out in paper [1]. I argued that a collision of two nuclei which are Lorentz-contracted into two plane sheets creates an environment for the perturbative regime of interaction between quarks and gluons, which was impossible before the collision: the perturbative vacuum itself is a product of the collision. Quantum dynamics of quarks and gluons before and after collision of the nuclei are qualitatively different. However, several important issues were left aside in the previous papers.

Quantum field theory has a strict definition of the *dynamics*. This notion was introduced by Dirac [3] at the end of 40-th in connection with his attempt to build a quantum theory of gravitational field. Every (Hamiltonian) dynamics includes its specific definition of the quantum mechanical observables on the (arbitrary) space-like surfaces, as well as the means to describe evolution of the observables from the "earlier" space-like surface to the "later" one. The "infinite momentum frame" (IMF) is a synonym of the "light-front dynamics," which had been introduced by Dirac as an example of dynamics , along with the other forms of dynamics. Besides the light-front dynamics, Dirac has also suggested the so-called point form of field dynamics which was conceived as a tool to describe the interaction of the field with the point-like classical particle.

Usually both nuclei before the collision are considered separately, in their own IMF, and we have two Hamiltonian dynamics with different definition of the time variable. This is a severe shortcoming of the theory. Factorization scheme suggests a detour. Instead of solving the problem, it replaces the true bound states of the quark and gluon fields in hadrons by the artificial flux of free partons. Factorization strictly requires a scale which is given by some *measured hard probe*, like high momentum transfer in DIS or the heavy dilepton mass in the Drell-Yan process. The method is not expected to work when the experiment does not suggest any "hard probe." Then the idea of factorization loses its footing and the formal factorization scale becomes an ill-defined infra-red cut-off. The heavy-ion collision is just the case. In fact, the nuclei probe each other at all scales and we are compelled to consider interaction of the two bounded systems without appealing to the parton picture [1]. Thus, it is imperative to find a way to describe quarks and gluons from both nuclei as well as the products of their interaction using *the same Hamiltonian dynamics*. This requirement follows solely from the fact that the precise definition of the field states (particles) depends on how the global observables are defined.

The proper definition of states is highly nontrivial and important question because the search of the QGP in heavy ion collision is, in the first place, the search for the evidence of the entropy production. Indeed, before the collision the quark and gluon fields are assembled into two coherent wave packets, the nuclei, and, therefore, the initial entropy equals zero. The coherence is lost and the entropy is created due to interaction. Though one may wish to rely on the

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invariant formula, $S = \mathrm{Sp}\rho \ln \rho$, which expresses the entropy S via the density matrix ρ, at least one basis of states should be found explicitly.

Amongst the all known solutions of physical problems, those which were starting with a lucky guess about the normal coordinates (degrees of freedom), are most elegant and successful. In QCD (and even in QED), an appropriate choice for the gluons (photons) is always difficult because the gauge is one of the elements of the Hamiltonian dynamics. The gauge, as the dynamics itself, is a global object, and both nuclei should be described using the same gauge condition. The light-front form of dynamics and the light-front gauge proved to be sensible tools to study the DIS process, the proton interaction with the structureless electron, basically because the gauge is physical. In this gauge, the interference of the final state gluons is suppressed and the ladder diagrams give the leading contribution to the DIS cross section.

The primary choice of the degrees of freedom is successful if, even without any interaction, dynamics of the normal modes adequately reflects main physical features of the phenomenon. Hence, the normal modes of the fields participating in the collision of two nuclei should be compatible with their Lorentz contraction. Unlike the incoming plane waves of the standard scattering theory, the nuclei have well defined shape and the space-time domain of their intersection is well defined. Of ten symmetries of the Poincare group, survive only rotation around the collision axis, boost along it, and the translations and boosts in the transverse directions. The idea of the two plane sheets collision immediately leads us to the *wedge form*; all possible states of quark and gluon fields before and after collision must be confined to within the past and the future light cones (wedges) with the (x, y) collision plane as the edge. It is profitable to limit in advance the set of normal modes to those which have the symmetry of the localized interaction. In this *ad hoc* approach, all the spectral components of the nuclear wave functions ought to collapse in the two-dimensional plane of interaction, even if all the confining interactions of quarks and gluons in the hadrons and the coherence of the hadronic wave functions are neglected.

In the wedge form of dynamics, the states of free quark and gluon fields are defined (normalized) on the space-like hypersurfaces of the constant proper time τ, $\tau^2 = t^2 - z^2$. The main idea of the approach is to study dynamical evolution of the interacting fields along the Hamiltonian time τ. The gauge of the gluon field is fixed by the condition $A^\tau = 0$. This simple idea solves several problems. On the one hand, it becomes possible to treat two different light-front dynamics which describe each nucleus of the initial state separately, as two limits of this single dynamics. On the other hand, after collision this gauge simulates a local temporal axial gauge. This feature provides a smooth transition to the boost-invariant regime of the created matter expansion (as the first approximation).

The feature of the states to collapse in the interaction vertex will be crucial for understanding the dynamics of the collision. A simplest optical prototype of the wedge dynamics is *camera obscura*, a dark chamber with the pin-hole in the wall. Amongst many possible *a priori* ways to decompose the incoming light, the camera selects one. Only centered at the pin-hole spherical harmonics can penetrate inside the camera. The spherical waves reveal the angular dependence at some distance from the center and build up the image on the opposite wall as a screen.

The dynamical example is reflection of light from the plane boundary between vacuum and the media. For monochromatic signal the solution of the problem is given by the Fresnel formulae which connect the amplitudes of the incoming, reflected and refracted waves. These triplets of waves can be viewed as the stationary states of the photons which obey the boundary condition. The latter depend on the frequency of the photon (due to dispersion of the refraction index) and on coordinate of the boundary (let it be the plane $z = 0$). Consider now the non-stationary process, when the electromagnetic wave has a front, $z - t = 0$, and is not monochromatic. The problem is linear and can be easily solved by presenting the light pulse as a superposition of the stationary solutions. Every stationary partial wave triplet knows nothing about its origin from the pulse. However, before the light front has reached the boundary $z = 0$, the reflected and refracted components of the triplets add up in such a way, that the refracted and reflected fields do not exist along with the field of the incoming wave ahead of the leading wave front. Mathematically, this is possible due to the proper balance of the amplitudes and phases in spectral decomposition of the signal. The first study of the wave front propagation in dispersive media was done by Sommerfeld and Brillouin at the beginning of the century [4] and had the goal to show that even in the case of anomalous dispersion, when the group velocity may exceed the velocity of light, the principles of special relativity are not violated. The full study of the transient process on the boundary of the media was done in Ref. [5].

The problem of the light front in the medium can be re-formulated in terms of quantum mechanics as well. The system of wave triplets may be quantized as any other system of the field oscillators, *etc.* Then emission of any new wave is just excitation of one of these oscillators or superposition of such excitations. There are infinitely many sets of wave triplets which correspond to the different positions of the boundary, $z = a > 0$, which may be used for spectral decomposition of the incoming wave at $z < 0$. The *incoming photon* can be considered as superposition of triplets from any of these sets since all these superpositions are virtual until the interaction of the wave with the media selects one of them as a consequence of the real process. Until a is an arbitrary parameter, these sets respect the translational symmetry of the free space in z-direction. Interaction of the wave front with the medium makes an explicit selection, $a = 0$, and destroys the initial symmetry. This simple example illustrates general principle: of

many possible decompositions of the initial state, real is the one, corresponding to actual interaction. The symmetry of the system, as it is seen before and after interaction, may be different.

Our further concern is localization of the interaction in collisions of the particles. This issue includes different questions. We say that electron, muon or quark are point-like since the lowest order vertex of their electromagnetic interaction does not require form-factor. However, this reflects only locality of the interaction between the fundamental fields of QED. Isolation of the domain in space and time where this local interaction happens is a quite separate question. Previous example tells us that the conclusive judgement about localization of interaction in the collision event can be made only *post factum*. Indeed, colliding two protons at very high energy, one cannot anticipate, how many particles will be produced in the final state. On average, the multiplicity is quite low, since the total cross section is always dominated by the soft peripheral processes. Only small fraction of events will have high multiplicity. This narrow subset consists of those were the geometry of interaction (transition currents) is effective enough with respect to excitation of a special set of modes. It is direct experimental evidence that at highest energies and highest multiplicities the spectrum of hadrons tends to have a plateau in the central rapidity region. Extrapolating this result to infinite energy, one immediately concludes that the clue to understanding this process is the Lorentz contraction of the colliding particles [6]. To produce the wide rapidity plateau, the system should lose any initial geometrical scale along with translational invariance in time direction and direction of the collision axis. The wedge form of dynamics has this *kinematic* property in its initial formulation and employs it as the classical boundary condition for the underlying dynamics of quantum fields. Phenomena with this kinematic regime can be met both in QED and QCD. However, only in QCD can we encounter the necessary ratio of scales: the typical scale of the vacuum fluctuations which leads to the confinement (*e.g.* size of the instanton) is of the order $0.3fm$, while already at $200\,A\,GeV$ nuclear collision the resolved longitudinal scale is $0.1fm$. Thus, there is a deep reason to rely on the wedge form of dynamics in most of the processes where the gluonic degrees of freedom are actively excited. A part of this excitation is creation of the new ground state, *the perturbative vacuum*. This state exists since the moment of first overlapping of the Lorentz-contracted hadrons and until hadronization completely washes it out at the proper time τ_{hadr} . Only after the hadronization process ends up, the system recovers its *translational invariance on hadronic scale*, and, if the rapidity plateau was infinite (in the limit of the infinite c.m.s energy), it never would. In fact, the intermediate perturbative dynamics of quark and gluon fields always takes place in the regime of the broken translational invariance!

By its logic, the wedge dynamics is contiguous both to the standard quantum scattering theory which relies on the parton picture and deals with the plane wave asymptotic states, and to the theory suggested by McLerran and Venugopalan [7–9], which starts with the picture of classical evolution of the gluon fields of colliding quark sources and treats quantum effects as small corrections to the classical background. However, there are several major differences.

(1) I do not view quarks as the sources of the gluon fields in hadrons or even in nuclei. Rather, I keep in mind the picture which is based on the long standing observation by Shuryak [10] that the energy scale of confinement is weak, while the energy density of QCD vacuum is very high. In his approach, the hadrons are thought of as the point-to-point correlators of various quark currents in physical vacuum and kept localized due to the tunneling. Recent development of this idea proved to be very successful in computation various properties of hadrons [11]. Though connection between the Euclidian regime of these calculations and Minkowskian dynamics of the real world is poorly understood, the above mentioned ratio of energy scales should be considered the most significant physical observation. In the advocated approach, the hadrons in the QCD vacuum can be compared to the sound waves in a heavy fluid; though the energy and momentum of the acoustic pulse may be very high, the atoms of the fluid experience very small displacements. However, nonlinear interaction of two waves may cause cavitation. To cause cavitation of the fluid by sound waves, one needs high gradients of the acoustic displacements (high tension) rater than high amplitude of the acoustic wave. I suggest to view the multiparticle production in hadronic collision like a signature of a similar phenomenon; "cavitation" of the physical QCD vacuum and creation of the domain with the perturbative dynamics of quarks and gluons. To switch it on, one needs extreme Lorentz contraction which resolves fluctuations of much shorter scale than those providing a natural "soft" confinement in the hadrons.

(2) Unlike in Refs. [7–9], I do not require any specific information about the wave functions of the hadrons or nuclei. Such a knowledge is needed to compute the hadronic or nuclear form-factors. In deeply inelastic processes one needs wave function only to support the concept of the finite size object and its Lorentz contraction. The structure functions of the deep inelastic processes reflect properties of the physical vacuum rather than of the hadrons.

(3) I view any deeply inelastic process as a transient process developing in space and time. I am interested primarily in the first few fermi of its history, before the asymptotic states of scattering are formed. During this period, longitudinal and transverse fields are not separated geometrically, as they never are in the IMF. In parallel studies I show that the spurious poles of the null-plane gauge carry indication of static configuration of the gauge field [12]. Analysis of the evolution equations for the observables of the inclusive process carried out in Ref. [13] shows that longitudinal and the transverse fields are not separated dynamically. Moreover, examination of the extended set of evolution equations for the structure functions of DIS brings to light a new element, the feed-back via longitudinal fields, which is consistent only with the power-like enhancement of the DIS cross section at low x. In the wedge

dynamics, which incorporates geometry of the DIS process already in the structure of states of free fields, longitudinal and transverse fields are not separated even mathematically [14].

(4) Analysis of Ref. [13] explicitly shows that an attempt to extend the IMF technique beyond the utilitarian needs of the parton model brings in severe singularities which are intrinsic for the IMF dynamics. Their physical regularization undermine status of x_F as physical variable. The wedge dynamics naturally cures this problem in such a way that *geometry of interaction* between the fields is *not* contracted into the singular geometry of plane sheets.

Since the wedge form of dynamics relies heavily on the localization of the interaction that physically resolve the QCD degrees of freedom, certain pre-requirements should be met. The idea of the point-like localization of the interaction is unambiguous for the finite-size objects like nuclei or hadrons in the environment of the process which resolve their internal structure. In the system of two nuclei, localization is, perhaps, perfect, and all dynamics takes place in the future wedge of the intersection plane.

Geometrical similarity of the *ep*– and *pp*–collisions is the major base for hope that the *ep*-DIS data contain the same information about the process of the proton destructure which is essential in the *pp*- and *AA*-collisions. The deeply inelastic interaction between the structureless electron and the hadron carries all necessary signs of localization which allow one to treat this process within the boundary conditions of the wedge dynamics. However, since QED has no intrinsic scale similar to hadronic scale Λ_{QCD} of quantum chromodynamics, one should be careful with an *a priori* localization of the structureless electron on the QCD scale, since this step would mean an assignment of the structure function to the electron itself! For this reason, I begin in section II with the discussion of how the wedge dynamics solves the problem of localization of the interaction. In section III, I discuss how the trigger in the measurement of the DIS cross section may affect the temporal order of the processes that takes place near the light cone. In Sec. IV and Sec. V I present Hamiltonian formalism for the scalar and the fermion fields in the wedge form of dynamics. The one-particle solutions for the free fields and their Wightman functions and propagators are obtained. These are necessary for the future formulation of the perturbation theory. Even for the free gauge field, calculation of the gluon modes and the gluon propagator is a complicated mathematical problem. Solution of this problem is described in the separate paper [14].

II. LOCALIZATION OF INTERACTIONS IN WEDGE DYNAMICS

For the further qualitative analysis of the approach it is enough to use the one-particle wave functions (4.11) of the scalar field. Let us write down the wave function $\Xi_{\theta,p_\perp}(x)$ in the following form:

$$\Xi_{\theta,p_\perp}(x) = \frac{1}{4\pi^{3/2}} e^{-im_\perp \tau \cosh(\eta-\theta)} e^{i\vec{p}_\perp \vec{r}_\perp} \quad . \tag{2.1}$$

This function describes a state of the scalar particle which occupies the future and the past light wedges of the collision plane. The above form implies that τ is positive in the future of the wedge vertex and negative in its past and, as usually, $m_\perp^2 = p_\perp^2 + m^2$. Though this wave function is an obvious plane wave, it carries the quantum number θ, rapidity of the particle, instead of the momentum p_z. The main physical difference is that the wave function (2.1) is normalized on the hypersurface $\tau = const$. At large $m_\perp|\tau|$, the phase of the wave function $\Xi_{\theta,p_\perp}$ is stationary in a very narrow interval around $\eta = \theta$ (outside this interval, the function reveals oscillations with exponentially increasing frequency); the wave function describes a particle with rapidity θ moving along the classical trajectory . However, for $m_\perp|\tau| \ll 1$, the phase is almost constant along the surface $\tau = const$. The smaller τ, the more uniformly the domain of stationary phase is stretched along the light cone. A single particle with the wave function $\Xi_{\theta,p_\perp}$ begins its life as the wave with the given rapidity θ at large negative τ. Later, it becomes spread over the boundary of the past light wedge at $\tau \to -0$. Being spread, it appears on the boundary of the future light wedge at $\tau \to +0$. Eventually, it again becomes a wave with rapidity θ at large positive τ. The size and location of the interval where the phase of the wave function is stationary will play a central role in all subsequent discussion, since it is equivalent to the localization. Indeed, overlapping of the domains of stationary phase in space and time provides the most effective interaction of the fields.

The size $\Delta\eta$ of the η-interval around the particle rapidity θ, where the wave function is stationary, is easily evaluated. Extracting from the exponential of Eq. (2.1) the trivial factor $e^{-im_\perp \tau}$ which defines evolution of the wave function in the τ-direction, we obtain an estimate:

$$2 \, m_\perp \tau \sinh^2(\Delta\eta/2) \sim 1 \quad . \tag{2.2}$$

The two limit cases are as follows,

$$\Delta\eta \sim \sqrt{\frac{2}{m_\perp \tau}}, \quad \text{at} \ \ m_\perp \tau \gg 1 \, , \quad \text{and} \quad \Delta\eta \sim \ln\frac{2}{m_\perp \tau}, \quad \text{at} \ \ m_\perp \tau \ll 1 \quad . \tag{2.3}$$

In the first case one may boost this interval into laboratory reference frame and see that the interval of stationary phase is Lorentz contracted (according to the rapidity θ) in z-direction.

Obviously, one may wish to deal with the wave packets built from the waves with different transverse momenta. The packets of the waves with the same rapidity θ do not experience dispersion in the longitudinal direction. [The language of the rapidity is extremely useful here since it supports an intuitive picture of the partial waves as co-movers which constitute the proton.] This kind of expansion can be employed to form the finite size objects. The amplitudes and the phases of the coefficients in this expansion are balanced in such a way that before the collision the wave packets represent the well shaped nuclei. If two localized objects simultaneously pass through the vertex, then the partial waves that form their wave functions effectively overlap in the vicinity of the light wedge. The interval of time, when the partial wave with transverse momentum p_t is spread, is of the order $\tau \sim 1/m_t$. The high-p_t components of the wave function assemble around the world line of the initial rapidity $\eta = \theta$ earlier, and have less time to interact than the low-p_t components. Perhaps, it is a good approximation to view the proton after it has passed the interaction vertex as a superposition of various plane waves with the same rapidity θ. If no interaction with the electron occurs at sufficiently small τ, then the amplitudes and the phases of these waves remain unchanged and the packet assembles into the initial proton. If only small fraction of partial waves with not too high p_t has interacted at not too small τ, then the entire process may become diffractive: the high-p_t components of the wave packet will assemble quite early along the proton's world line and build up a core of a slightly disturbed proton.

In general, Lorentz-contraction is a feature of the finite size objects. However, it is well known that quantum cross sections of the Born's approximation coincide with those obtained classically. Classical treatment explicitly accounts for the Lorentz transformation of the fields of the colliding particles. It alone leads to the localization of the interaction and builds up a geometrical picture of the plane sheets. Quantum mechanics deals with interaction of the waves and does not support the image of the Lorentz contracted classical field. Therefore, the physical localization of the interaction in the wave dynamics requires localization of the transition currents which is provided by the wave functions.

Let the particle has rapidities θ and θ' and transverse masses $m_\perp$ and $m'_\perp$ before and after scattering, respectively. The change of its state is due to the transition current,

$$j_{\mathbf{k},\mathbf{k}'}(x_1) \sim e^{-i\tau[m_\perp - m'_\perp]} e^{-i\tau[m_\perp \sinh^2 \frac{\theta - \eta_1}{2} - m'_\perp \sinh^2 \frac{\theta' - \eta_1}{2}]} \quad . \qquad (2.4)$$

For the process with high transverse momentum transfer we have $m'_\perp \gg m_\perp$. It is easy to see that at large difference $\Delta\theta = \theta - \theta'$ the stationary phase intervals of the two modes overlap only at sufficiently small τ, when $m_\perp \tau \sim e^{-\Delta\theta/2}$. At this moment, the long stationary phase domain of the incoming particle ($m_\perp = m$ is low) begins to cover still narrow stationary phase domain of the scattered particle ($m'_\perp$ is high). Similar analysis is valid for the transition current of the second particle, $j_{\mathbf{p},\mathbf{p}'}(x_2)$. Distance between the points x_1 and x_2 along the surface $\tau = const$ is $R = [(\vec{r}_{\perp 1} - \vec{r}_{\perp 2})^2 + \tau^2 \sinh^2(\eta_1 - \eta_2)]^{1/2}$. The Coulomb part of the vector field propagator in the gauge $A^\tau = 0$ is proportional to R^{-2} [14]. At very small τ, even at finite rapidity difference $\Delta\theta$, the distance between transition currents $j_{\mathbf{k},\mathbf{k}'}(x_1)$ and $j_{\mathbf{p},\mathbf{p}'}(x_2)$ is entirely due to their transverse separation, $|\vec{r}_{\perp 1} - \vec{r}_{\perp 2}|$. Therefore, the process of scattering indeed takes place in the vicinity of the wedge vertex, in compliance with the classical treatment. Moreover, interaction that excites mode with rapidity θ' and transverse momentum $k'_\perp$ takes place at small $\tau \sim 1/k'_\perp$ at some interval of η around θ'. It virtue of (2.3), the higher $k'_\perp$, the narrower this interval is. By the current conservation, both initial- and final-state wave functions should effectively overlap on this interval.

The last conclusion is in strong correlation with an observation stimulated by the study of the gluon correlator in the gauge $A^\tau = 0$ [14]. The limit of the gluon propagator of this gauge near the null-plane $x^- = 0$ is the propagator of the gauge $A^- = 0$. Close to the plane $x^+ = 0$ the same propagator corresponds to the gauge $A^+ = 0$. The gauge $A^+ = 0$ is obviously used to describe the QCD evolution of the proton with momentum $P^+ \to \infty$ and $P^- = 0$ which enters into collision with the electron along the null-plane $x^- = 0$. However, the gluon propagator of the gauge $A^+ = 0$ is found near the opposite boundary of the light wedge, where, according to the wedge dynamics, the electromagnetic transition current extinguishes the old wave function of the quark which was "prepared" for the last interaction by the QCD evolution, and excites the new one, with the highest $p_\perp^2 \sim Q^2$. This is possible only if at least the latest stage of the QCD evolution takes place near the null-plane $x^+ = 0$! Thus, the existing theory of QCD evolution seems to support the geometry of high-energy collision suggested by the wedge form of dynamics.

What happens at other rapidities, where the transition currents are not stationary? Though the currents experience rapid oscillations in η-direction, and conditions for the high-p_t jet formation are not fulfilled, the wave functions overlap there as well. Being unable to produce the coherent jet, these fragments of the wave functions should break up into many soft quanta which, either uniformly or with gaps, fill in the rapidity interval between the leading jets.

We arrived at the major conclusion that at high relative rapidity of the colliding particles the most effective interaction takes place in the vicinity of the light cone. This conclusion is very close to the observation made by Ioffe long ago, that the lepto-production process is dominated by the close vicinity of the light cone and long distances in

the direction of the virtual photon propagation [15]. However there is conceptual difference between the Ioffe's and the present motivations of this picture.

The first step of Ioffe's approach is to use the translation invariance and to replace the product $j^\mu(x)j^\nu(y)$ of two hadronic currents in the matrix element of the inclusive process by their commutator $[j^\mu(x), j^\nu(y)]$. By causality, the latter vanishes at space-like separation between x and y. The second step is to employ the Bjorken scaling as a pre-requirement.

The first of these steps is somewhat ambiguous since the momentum transfer q^μ in the lepto-production is space-like, $q^2 < 0$. Therefore, the required inequality, $q^0 > 0$, is not Lorentz invariant and the replacement of product of the two currents, jj, by their commutator, $[j, j]$, is possible only in a restricted kinematic region or in a special reference frame. The proximity to the light cone in the lepto-production process, as it emerges from this derivation, does not look (unlike the light cone itself) as a Lorentz-invariant feature. However, the plateau in the rapidity distribution of hadrons at extreme c.m.s. energies says that it does. The wedge form of dynamics avoids the reference frame dependent arguments. The Ioffe's long distances are replaced by the large intervals of the rapidity coordinate η. The entire picture can be boosted from one reference frame to another without any changes in physical interpretation or qualitative change of the normal modes.

III. TEMPORAL ORDER IN INCLUSIVE PROCESSES

The program of computing the quark and gluon distributions in heavy ion collisions relies on the data obtained in seemingly more simple processes, like ep-DIS, *etc.* It turns out that differently triggered sets of data may carry significantly different information. In this section, we discuss two examples and show that even minor change of the way to take the data may strongly affect the type of the studied process.

The early discussion of the role of the light cone distances in the high energy collisions [16] resulted in Gribov's idea of the two-step treatment of the inelastic processes [17]: the gamma-quantum *first* decays into virtual hadrons and *later* these hadrons interact with the nuclear target. This idea looks very attractive since it explains the origin of two leading jets in electro-production events, corresponding to the target and the projectile (photon) fragmentation. It provides reasonable explanation of the plateau in the rapidity distribution of the hadrons also. However, this elegant qualitative picture contains a disturbing element, the way how the words "first" and "later" are used.

The question of temporal sequence in relativistic quantum mechanics is two-fold. The first issue is trivial; any statement concerning the time ordering or causality should respect the light cone boundaries. The second one concerns the nature of quantum-mechanical evolution which allows one to read out any dynamical information only after the evolution is interrupted by the measurement. This feature is usually referred to as a collapse of the wave function. All dynamics of the system takes place before the measurement. It was already pointed out [1] that the technique of QFK explicitly supports this principle. The Gribov's idea of the initial fragmentation of the γ^* in lepto-production process contradicts it. Indeed, the only quantity measured in the deep inelastic scattering experiment is the momentum of the final state electron. Therefore, the process of the momentum transfer to the electron must be the last one in temporal sequence. The off-mass-shell γ-quantum which provides this transfer must be a product of the preceeding evolution of the hadronic degrees of freedom. Below, we shall show how this result emerges in the QFK version of the S-matrix approach.

The Gribov's picture will be shown to describe different class of experiments, where the final state electron is entirely off the control and the event is triggered by the high p_t quark or gluon jet in the final state. In fact, the two leading jets corresponding to the target and the projectile fragmentation are present in both types of events, tagged either by the high p_t electron or by the high p_t jet. Regardless the dynamical details, the hadronic distribution is driven by the geometry of the transition currents near the light wedge of the interaction plane.

Let us consider collision process with parameters of only one final state particle explicitly measured. Let this particle be the electron with momentum $\mathbf{k}'$ and spin σ'. The deep inelastic electron-proton scattering is an example of such experiment. All vectors of final states which are accepted into the data ensemble are of the form $a^\dagger_{\sigma'}(\mathbf{k}')|X\rangle$ where the vectors $|X\rangle$ form a full set. The initial state consists of the electron with momentum $\mathbf{k}$ and spin σ and some other particle or composite system carrying quantum numbers P. Thus the initial state vector is $a^\dagger_\sigma(\mathbf{k})|P\rangle$. The inclusive transition amplitude reads as $\langle X|a_{\sigma'}(\mathbf{k}') \, S \, a^\dagger_\sigma(\mathbf{k})|P\rangle$ and the inclusive momentum distribution of the final state electron is the sum of the squared moduli of these amplitudes over the full set of the non-controlled states $|X\rangle$. We obtain the following formula,

$$\frac{dN_e}{d\mathbf{k}'} = \langle P|a_\sigma(\mathbf{k})S^\dagger a^\dagger_{\sigma'}(\mathbf{k}')a_{\sigma'}(\mathbf{k}')Sa^\dagger_\sigma(\mathbf{k})|P\rangle \; , \tag{3.1}$$

which is just an average of the Heisenberg operator of the number of the final state electrons over the initial state. Since state $|P\rangle$ contains no electrons, one may commute electron creation and annihilation operators with the S-

matrix and its conjugate $S^\dagger$ pulling the Fock operators a and $a^\dagger$ to the right and to the left respectively. Let $\psi^{(+)}_{\mathbf{k}\sigma}(x)$ be the one-particle wave function of the electron. Then the procedure results in

$$\frac{dN_e}{d\mathbf{k}'} = \frac{1}{2}\sum_{\sigma\sigma'}\int dxdx'dydy'\overline{\psi}^{(+)}_{\mathbf{k}\sigma}(x)\overline{\psi}^{(+)}_{\mathbf{k}'\sigma'}(x')\langle P|\frac{\delta^2}{\delta\overline{\Psi}(x)\delta\Psi(y)}\left(\frac{\delta S^\dagger}{\delta\Psi(y')}\frac{\delta S}{\delta\overline{\Psi}(x')}\right)|P\rangle\psi^{(+)}_{\mathbf{k}\sigma}(y)\psi^{(+)}_{\mathbf{k}'\sigma'}(y') \tag{3.2}$$

Introducing the Keldysh convention about the contour ordering [18,2], we may rewrite this expression as

$$\frac{dN_e}{d\mathbf{k}'} = \frac{1}{2}\sum_{\sigma\sigma'}\sum_{AB}(-1)^{A+B}\int dxdx'dydy'\overline{\psi}^{(+)}_{\mathbf{k}\sigma}(x)\overline{\psi}^{(+)}_{\mathbf{k}'\sigma'}(x')\langle P|\frac{\delta^4 S_c}{\delta\overline{\Psi}(x_A)\delta\overline{\Psi}(y'_1)\delta\Psi(x'_0)\delta\Psi(y_B)}|P\rangle\psi^{(+)}_{\mathbf{k}\sigma}(y)\psi^{(+)}_{\mathbf{k}'\sigma'}(y') \tag{3.3}$$

where $S_c = S^\dagger S$. The electron couples only to the electromagnetic field. Therefore, to the lowest order,

$$\frac{dN_e}{d\mathbf{k}'} = \frac{1}{2}\sum_{\sigma\sigma'}\int dxdy\overline{\psi}^{(+)}_{\mathbf{k}'\sigma'}(y)\overline{\psi}^{(+)}_{\mathbf{k}\sigma}(x)\langle P|\ \mathbf{A}(x)\ \mathbf{A}(y)|P\rangle\psi^{(+)}_{\mathbf{k}\sigma}(y)\psi^{(+)}_{\mathbf{k}'\sigma'}(x)\ \ , \tag{3.4}$$

where $\mathbf{A}(x)$ is the Heisenberg operator of the electromagnetic field. Already at this very early stage of calculations, the answer has a very clear physical interpretation. Once only the final state electron is measured, the probability of the electron scattering is entirely defined by the electromagnetic field produced by the rest of the system evolved from its initial state till the moment of interaction with the electron. The trigger for this measurements is presence of the electron with high enough transverse momentum among the secondaries.

We may obtain the answer either using equations of QFK [2,1], or iterating Eq. (3.4) by means of the Yang-Feldman equation, $\mathbf{A}(x) = \int d^4 y D_{ret}(x,y)\mathbf{j}(y)$, where $\mathbf{j}(y)$ is the Heisenberg operator of electromagnetic current and $D_{ret}(x,y)$ is the retarded propagator of the photon. Summation over the electrons spins invokes the leptonic tensor $L_{\mu\nu}(k,k')$. If $q = k - k'$ is the space–like momentum transfer, then the DIS cross-section is given by

$$k'_0\frac{d\sigma}{d\mathbf{k}'} = \frac{i\alpha}{(4\pi)^2}\frac{L_{\mu\nu}(k,k')}{(kP)}\Delta_{ret}(q)W^{\mu\nu}(q)\Delta_{adv}(q)\ \ , \tag{3.5}$$

where $W^{\mu\nu}(q)$ is the standard Bjorken notation for the correlator of two electromagnetic currents,

$$W^{\mu\nu}(q) = \frac{2V_{lab}P^0}{4\pi}\langle P|\mathbf{j}^\mu(x)\mathbf{j}^\nu(y)|P\rangle\ \ . \tag{3.6}$$

where V_{lab} and P^μ are the normalization volume and the momentum of the proton in the laboratory frame. Correlator of the currents is the source of the field which has scattered the electron. Here, both photon propagators are retarded and respect the causal order of the process. Since the transition currents in the interaction of the ultrarelativistic particles are confined to the nearest vicinity of the light cone, the simplest geometric arguments support the picture when the last process of the electron scattering is due to the transition current which acts in the future of the wedge vertex.

Any details of the processes that take place in sector of strong interaction remain unobserved in this type of measurement. The observed ones are only absorbtion of the electron from the initial state and excitation the final state mode of the electron field which take place with respect to the normal translation-invariant QED vacuum and should not be localized. It is the hadronic plateau which indicates that the electromagnetic transition current of the strongly interacting quarks was localized together with all quark-gluon dynamics. The reason to treat QED and QCD vacua in so different manner is that of these two, only QCD has an intrinsic scale, Λ_{QCD}.

What happens if we change the observable in the same deep inelastic process initiated by the *ep*-interaction. Let us trigger events on the high-p_t quark or gluon jet in the final state regardless the momentum of the electron. Then the data ensemble includes the states $\alpha_{\sigma'}(\mathbf{p})|X\rangle$, with the inclusive quark jet, where $\alpha_\sigma(\mathbf{p})$ is the Fock operator for the final state quark. The inclusive amplitude for this process is $\langle X|\alpha_{\sigma'}(\mathbf{p})\ S\ a^\dagger_\sigma(\mathbf{k})|P\rangle$. To produce the quark, the hadronic target has to be hit either by the photon coming from the first step of virtual fragmentation of the electron or by the partons, the electron has previously fragmented into. Now we encounter quite different type of fluctuations and both initial electron and the proton should be treated in the wedge form of dynamics. Thus, the change of the trigger drastically affects information read out of the data. Trigger on the high-p_t jet in the same process allows one to filter out fluctuations corresponding to the Gribov's picture.

This type calculations, extended to the higher orders in coupling [1] have lead to the conclusion that the QCD evolution describe (in a real time scale) the gradual process of excitation a special wave packet which is "resonant" to the interaction in a given inclusive measurement . Within the wedge form of dynamics one can learn more about where the process takes place in space and time.

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IV. STATES OF SCALAR PARTICLES PARTICIPATING THE PLANAR INTERACTION

From the physical motivation of the previous section it is clear that the wedge form of dynamics will require special tools for the consistent development of its mathematical formalism. To begin with, I shall discuss the wedge form of the dynamics of the charged scalar particles. They are not expected to participate in the heavy ion collision at its early stage. However, it is instructive to work out this simple case since its mathematics is very simple and allows one to illustrate the main ideas without extraneous details.

A. The classical treatment

Let us assume that the collision occurs in the plane $t = 0$, $z = 0$. Two planes, $t = z$ and $t = -z$, divide the whole space-time into four domains, future (F), past (P), left (L) and right (R) with respect to the collision plane. In these domains we shall use the following coordinates:

$$F: \quad t = \tau \cosh \eta, \ z = \tau \sinh \eta, \qquad P: \quad t = -\tau \cosh \eta, \ z = -\tau \sinh \eta,$$
$$L: \quad t = -\tau \sinh \eta, \ z = -\tau \cosh \eta, \quad R: \quad t = \tau \sinh \eta, \quad z = \tau \cosh \eta \ . \tag{4.1}$$

These coordinates induce the metric, different in the different domains,

$$(FP): ds^2 = d\tau^2 - dx^2 - dy^2 - \tau^2 d\eta^2, \quad (LR): ds^2 = \tau^2 d\eta^2 - dx^2 - dy^2 - d\tau^2 \ . \tag{4.2}$$

Any normal field theory begins with the action. For the complex scalar field the action is

$$\mathcal{A} = \int d^4x \sqrt{-g}\mathcal{L}(x) = \int d^4x \sqrt{-g}[g^{\mu\nu}\partial_\mu \phi^*(x)\partial_\nu \phi(x) - m^2 \phi^*(x)\phi(x)] \ . \tag{4.3}$$

Variation of the action with respect to the fields ϕ and ϕ^* yields the Lagrangian equations of motion,

$$\partial_\mu[(-g)^{1/2}g^{\mu\nu}(x)\partial_\nu \phi(x)] + (-g)^{1/2}m^2\phi(x) = 0, \quad \partial_\mu[(-g)^{1/2}g^{\mu\nu}(x)\partial_\nu \phi^*(x)] + (-g)^{1/2}m^2\phi^*(x) = 0 \tag{4.4}$$

and invoke the locally conserved $U(1)$–current,

$$J_\mu(x) = i\phi^*(x)\overleftrightarrow{\partial}_\mu\phi(x), \quad (-g)^{-1/2}\partial_\mu[(-g)^{1/2}g^{\mu\nu}(x)J_\nu(x)] = 0 \tag{4.5}$$

We shall start constructing the one-particle solutions from the F-domain, were the final states are supposed to be localized. Here, the corresponding wave functions of the free scalar particles obey the Klein-Gordon equation,

$$\frac{1}{\tau}\frac{\partial}{\partial\tau}(\tau\frac{\partial\phi}{\partial\tau}) - \frac{1}{\tau^2}\frac{\partial^2\phi}{\partial\eta^2} - \nabla_\perp^2\phi + m^2\phi = 0 \ , \quad \frac{1}{\tau}\frac{\partial}{\partial\tau}(\tau\frac{\partial\phi^*}{\partial\tau}) - \frac{1}{\tau^2}\frac{\partial^2\phi^*}{\partial\eta^2} - \nabla_\perp^2\phi^* + m^2\phi^* = 0 \ . \tag{4.6}$$

These equations remain unchanged for the P-domain and their first two terms change their sign in the domains L and R. At $t^2 - z^2 > 0$, these equations can be alternatively obtained as the equations of the Hamiltonian dynamic along the proper time τ. The canonical momenta conjugated to the fields ϕ and ϕ^* are

$$\pi_\phi(x) = \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta\dot\phi(x)} = \tau\dot\phi^*(x) \quad \text{and} \quad \pi_{\phi^*}(x) = \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta\dot\phi^*(x)} = \tau\dot\phi^*(x) \ , \tag{4.7}$$

respectively. The Hamiltonian of the field is as follows,

$$H = \int \tau d\eta d^2\mathbf{r} \ [\tau^{-2}\pi_{\phi^*}\pi_\phi + \tau^{-2}\partial_\eta\phi^*\partial_\eta\phi + \partial_x\phi^*\partial_x\phi + \partial_y\phi^*\partial_y\phi] \ , \tag{4.8}$$

and the wave equations are just the Hamiltonian equations of motion for the momenta.

The explicit form of the one-particle wave functions for the scalar particles and anti-particles is as follows,

$$\xi_{\nu,\vec{p}}^{(\pm)}(x) = \frac{e^{-\pi\nu/2}}{2^{5/2}\pi}H_{\mp i\nu}^{(2)}(m_\perp\tau)e^{\mp i\nu\eta}e^{\pm i\vec{p}\vec{r}} \ . \tag{4.9}$$

where $\vec{p} \equiv \vec{p}_\perp = (p_x, p_y)$, and $m_\perp^2 = p_\perp^2 + m^2$. In agreement with the chosen dynamics, the eigenfunctions $\xi_{\nu,p_\perp}(x)$ are normalized on the space-like hypersurfaces $\tau = const$ within the future light wedge of the collision point:

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$$(\xi^{(\pm)*}_{\nu,p_\perp}, \xi^{(\pm)}_{\nu',p'_\perp}) = \int \tau \, d\eta \, d^2\vec{r} \, \xi^{(\pm)*}_{\nu,p_\perp}(x) \, i\frac{\overleftrightarrow{\partial}}{\partial\tau} \, \xi^{(\pm)}_{\nu',p'_\perp}(x) = \delta(\nu - \nu')\delta(\vec{p}_\perp - \vec{p}'_\perp), \quad (\xi^{(\pm)*}_{\nu,p_\perp}, \xi^{(\mp)}_{\nu',p'_\perp}) = 0. \qquad (4.10)$$

This norm is a consequence of the equation (4.5) which expresses the current conservation. To obtain it, one needs to integrate (4.5) over the 4-volume and transform the volume integral to the integral over the closed surface. Thus we may need to continue the solution found in the F-domain to the domains P, R and L. First, let us notice that the wave packets,

$$\Xi^{(\pm)}_{\theta,p_\perp}(x) = \frac{\mp i}{(2\pi)^{1/2}} \int_{-\infty}^{+\infty} d\nu e^{\mp i\nu\theta} \xi^{(\pm)}_{\nu,p_\perp}(x) = \frac{1}{4\pi^{3/2}} e^{\mp ip^0 t + ip^z z} e^{\pm i\vec{p}_\perp \vec{r}_\perp} \, , \qquad (4.11)$$

can be easily recognized as the plane waves confined to within the F-domain. The r.h.s. of the Eq.(4.11) readily continue the solutions to all (t,z)-plane. Performing the inverse transformation, one obtains the following expressions,

$$\xi^{(\pm,F)}_{\nu,\vec{p}}(x) = \xi^{(\mp,P)}_{\nu,\vec{p}}(x) = \frac{e^{-\pi\nu/2}}{2^{5/2}\pi} H^{(2)}_{\mp i\nu}(m_\perp \tau) e^{\mp i\nu\eta} e^{\pm i\vec{p}\vec{r}}, \qquad (4.12)$$

$$\xi^{(\pm,L)}_{\nu,\vec{p}}(x) = \xi^{(\mp,R)}_{\nu,\vec{p}}(x) = \frac{\pm i e^{-\pi\nu/2}}{2^{3/2}\pi^2} K_{i\nu}(m_\perp \tau) e^{\mp i\nu\eta} e^{\pm i\vec{p}\vec{r}} \, . \qquad (4.13)$$

In what follows, I shall denote this piecewise-defined function by one symbol $\xi^{(\pm)}_{\nu,p_\perp}(x)$. Using Eqs.(4.13) one can explicitly check that at the time-like hypersurfaces $\tau = const$ in the domains L and R the normal flux of the charge vanishes locally (since $K_{i\nu}(m_\perp \tau) = K_{-i\nu}(m_\perp \tau)$ is a real function); the states prepared via the initial data at the space-like hypersurface $\tau = const$ in the P-domain are predetermined to penetrate the future light cone through its vertex. The solutions of the relativistic field equations are allowed to have discontinuities along the light cone characteristics. Therefore, the problems with strong localization of the interaction domain in space and time, are of special kind. Placing the vertex of the light cone (wedge) in the vertex of interaction, we can discard any possible continuation of the partial waves out of the P- and F-domains.

The wave functions $\xi^{(\pm)}_{\nu,p_\perp}(x)$ given by Eqs. (4.12) and (4.13) can be easily modified in such a way that the coordinates of the wedge vertex will be an additional parameter. Evidently, the system of these one particle solutions, with the arbitrary coordinates of the vertex, is isomorphic to the genuine system (4.11) of the plane waves $\Xi^{(\pm)}_{\theta,p_\perp}(x)$. Therefore, with the vertex coordinates explicitly retained, the extended set keeps the translational symmetry inherent to the full set of the non-triggered events.

The functions $\xi_{\nu,p_\perp}(x)$ can be employed for decomposition of the field:

$$\phi(x) = \int d^2\vec{r} d\nu [a_{\nu,p_\perp} \xi^{(+)}_{\nu,p_\perp}(x) + b^\dagger_{\nu,p_\perp} \xi^{(-)}_{\nu,p_\perp}(x)] \, , \quad \phi^\dagger(x) = \int d^2\vec{r} d\nu [a^\dagger_{\nu,p_\perp} \xi^{(+)*}_{\nu,p_\perp}(x) + b_{\nu,p_\perp} \xi^{(-)*}_{\nu,p_\perp}(x)] \, , \qquad (4.14)$$

where we have passed to the symbolics which will become relevant after the quantization. This symbolics implies that we have already gave the definitions of the particle and anti-particle states.

The plane waves (4.11) can be equally used to decompose the field operators,

$$\phi(x) = \int d^2\vec{p}_\perp d\theta [\alpha_{\theta,p_\perp} \Xi^{(+)}_{\theta,p_\perp}(x) + \beta^\dagger_{\theta,p_\perp} \Xi^{(-)}_{\theta,p_\perp}(x)], \quad \phi^\dagger(x) = \int d^2\vec{p}_\perp d\theta [\alpha^\dagger_{\theta,p_\perp} \Xi^{(+)*}_{\theta,p_\perp}(x) + \beta_{\theta,p_\perp} \Xi^{(-)*}_{\theta,p_\perp}(x)], \qquad (4.15)$$

thus providing the common language of the momentum decomposition. In our case this language is somewhat restricted because in the geometry of localized interaction we miss the quantum operator of the conserved momentum.

However, mathematically, we deal with the solutions of the homogeneous wave equations in free space. Thus the correlators remain unchanged. For example, in virtue of $d\theta = dp_z/p^0$, we have

$$D_{10}(x,y) = -i\langle 0|\phi(x)\phi^*(y)|0\rangle = -i \int d\theta d^2\vec{p}_\perp \Xi^{(+)}_{y,p_\perp}(x) \Xi^{(+)*}_{y,p_\perp}(y) \, ,$$

$$D_{01}(x,y) = -i\langle 0|\phi^*(y)\phi(x)|0\rangle = -i \int d\theta d^2\vec{p}_\perp \Xi^{(-)}_{y,p_\perp}(x) \Xi^{(-)*}_{y,p_\perp}(y) \, . \qquad (4.16)$$

In the momentum representation we have,

$$D_{\substack{10\\01}}(x,y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} [-2\pi i\delta(p^2 - m^2)\theta(\pm p^0)] \, . \qquad (4.17)$$

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One may easily recognize the standard expression for $D_{10}(p)$ and $D_{01}(p)$ in the integrand. The latter represents the density of states for the final-state particles and anti-particles in the initially unoccupied vacuum. Dependence of $D_{10}(x,x')$ and $D_{01}(x,x')$ only on the difference $x-x'$ is fully consistent with the free field dynamics.

These two Wightman functions lead to a familiar form of various propagators. Introducing the commutator $D_0 = D_{10} - D_{01}$, we have in coordinate form:

$$D_{\substack{ret\\adv}}(x,x') = \theta[\pm(x_0 - x_0')]D_0(x - x') \, ,$$

$$D_{00}(x,x') = -i\langle 0|T\phi(x)\phi^*(y)|0\rangle = \theta(x_0 - x_0')D_{10}(x,x') + \theta(x_0' - x_0)D_{01}(x,x') \, ,$$

$$D_{11}(x,x') = -i\langle 0|T^\dagger\phi(x)\phi^*(y)|0\rangle = \theta(x_0' - x_0)D_{10}(x,x') + \theta(x_0 - x_0')D_{01}(x,x') \, , \tag{4.18}$$

so that in momentum representation we have

$$D_{\substack{ret\\adv}}(p) = \frac{1}{(p_0 \pm i0)^2 - \mathbf{p}^2 - m^2} \, , \quad D_{\substack{00\\11}}(p) = \frac{\pm 1}{p_0^2 - \mathbf{p}^2 - m^2 \pm i0} \, . \tag{4.19}$$

Since the commutator $D_0(x,x')$ vanishes outside the light cone, the theta-functions in the definition of the retarded and advanced propagators can be conveniently rewritten as $\theta[\pm(\tau - \tau')]$. In virtue of $D_{00} = D_{ret} + D_{01}$ and $D_{11} = D_{10} - D_{ret}$, the other theta-functions acquire the same invariant definitions.

B. Quantization

The canonical commutation relations between the field coordinates and momenta ϕ and π_ϕ, ϕ^* and π_{ϕ^*}, read as

$$\tau[\phi(\eta, \vec{r}), \dot{\phi}^*(\eta', \vec{r'}] = \delta(\eta - \eta')\delta(\vec{r} - \vec{r'});$$

$$\tau[\phi^*(\eta, \vec{r}), \dot{\phi}(\eta', \vec{r'}] = \delta(\eta - \eta')\delta(\vec{r} - \vec{r'}). \tag{4.20}$$

They are satisfied if and only if the commutation relations between the Fock operators are of the standard form,

$$[a_{\nu,p_\perp}, a^\dagger_{\nu',\vec{p'}}] = [b_{\nu,p_\perp}, b^\dagger_{\nu',\vec{p'}}] = \delta(\nu - \nu')\delta(\vec{p} - \vec{p'}), \tag{4.21}$$

and

$$[\alpha_{\theta,\vec{p}}, \alpha^\dagger_{\theta',\vec{p'}}] = [\beta_{\theta,\vec{p}}, \beta^\dagger_{\theta',\vec{p'}}] = \delta(\theta - \theta')\delta(\vec{p} - \vec{p'}) \tag{4.22}$$

V. STATES OF FERMIONS

The states of fermions are studied along the same lines as the states of the bosons. A minor complication is due to the nontrivial form of the covariant derivative of the spinor in the curvilinear coordinates. The spinor field is essentially defined in the tangent space and, therefore, its covariant derivative should be calculated in the so-called tetrad formalism [19,20].

The covariant derivative of the tetrad vector includes two connections (gauge fields). One of them, $\Gamma^\lambda_{\mu\nu}(x)$, is the gauge field which provides covariance with respect to the general transformation of coordinates. The second gauge field, $\omega_\mu^{ab}(x)$, the spin connection, provides covariance with respect to the local Lorentz rotation.

Let $x^\mu = (\tau, x, y, \eta)$ be the contravariant components of the curvilinear coordinates and $x^a = (t, x, y, z) \equiv (x^0, x^1, x^2, x^3)$ those of the flat Minkowsky space. Then the tetrad vectors e^a_μ can be taken as follows,

$$e^0_\mu = (1, 0, 0, 0), \quad e^1_\mu = (0, 1, 0, 0), \quad e^2_\mu = (0, 0, 1, 0), \quad e^3_\mu = (0, 0, 0, \tau) \, . \tag{5.1}$$

They correctly reproduce the curvilinear metric $g_{\mu\nu}$ and the flat Minkowsky metric g_{ab}:

$$g_{\mu\nu} = g_{ab}e^a_\mu e^b_\nu = \text{diag}[1, -1, -1, -\tau^2] \, , \quad g^{ab} = g^{\mu\nu}e^a_\mu e^b_\nu = \text{diag}[1, -1, -1, -1] \, . \tag{5.2}$$

The spin connection can be found from the condition that the covariant derivatives of the tetrad vectors equal to zero,

$$\nabla_\mu e^a{}_\nu = \partial_\mu e^a{}_\nu + \omega_\mu{}^a{}_b e^b{}_\nu - \Gamma^\lambda{}_{\mu\nu} e^a{}_\lambda = 0 \quad, \tag{5.3}$$

$$\Gamma^\lambda{}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho} \left[\frac{\partial g_{\rho\mu}}{\partial x^\nu} + \frac{\partial g_{\rho\nu}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\rho} \right] \quad.$$

The covariant derivative of the spinor field includes only the spin connection,

$$\nabla_\mu \psi(x) = \left[\partial_\mu + \frac{1}{4} \omega_\mu{}^{ab}(x) \Sigma_{ab} \right] \psi(x) \ , \tag{5.4}$$

where $\Sigma^{ab} = \frac{1}{2}[\gamma^a \gamma^b - \gamma^b \gamma^a]$ is an obvious generator of the Lorentz rotations and γ^a are the Dirac matrices of Minkowsky space. Introducing the Dirac matrices in curvilinear coordinates, $\gamma^\mu(x) = e^\mu{}_a(x)\gamma^a$, one obtains the Dirac equation in curvilinear coordinates,

$$[\gamma^\mu(x)(i\nabla_\mu + gA_\mu(x)) - m]\psi(x) = 0 \ , \tag{5.5}$$

where $A^\mu(x)$ is the gauge field associated with the local group of the internal symmetry. The conjugated spinor is defined as usually, $\overline{\psi} = \psi^\dagger \gamma^0$, and obeys the equation,

$$(-i\nabla_\mu + gA_\mu(x))\overline{\psi}(x)\gamma^\mu(x) - m\overline{\psi}(x) = 0 \quad. \tag{5.6}$$

These two Dirac equations correspond to the action,

$$\mathcal{A} = \int d^4x \sqrt{-g} \mathcal{L}(x) = \int d^4x \sqrt{-g} \{ \frac{i}{2} [\overline{\psi}\gamma^\mu(x)\nabla_\mu\psi - (\nabla_\mu\overline{\psi})\gamma^\mu(x)\psi] + g\overline{\psi}\gamma^\mu(x)A_\mu\psi - m\overline{\psi}\psi \} \quad, \tag{5.7}$$

from which one easily obtains the locally conserved $U(1)$-current,

$$J^\mu(x) = \overline{\psi}(x)\gamma^\mu(x)\psi(x) \ , \quad (-g)^{-1/2}\partial_\mu[(-g)^{1/2}g^{\mu\nu}(x)J_\nu(x)] = 0 \ . \tag{5.8}$$

The Dirac equations (5.5) and (5.6) can be alternatively obtained as the equations of the Hamiltonian dynamics along the proper time τ. The canonical momenta conjugated to the fields ψ and $\overline{\psi}$ are

$$\pi_\psi(x) = \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta\dot{\psi}(x)} = \frac{i\tau}{2}\overline{\psi}(x)\gamma^0 \ \text{ and } \ \pi_{\overline{\psi}}(x) = \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta\dot{\overline{\psi}}(x)} = -\frac{i\tau}{2}\gamma^0\psi(x) \ , \tag{5.9}$$

respectively. The Hamiltonian of the Dirac field in the wedge dynamics has the following form,

$$H = \int \tau d\eta d^2\vec{r} \sqrt{-g} \{ -\frac{i}{2}[\overline{\psi}\gamma^i(x)\nabla_i\psi - (\nabla_i\overline{\psi})\gamma^i(x)\psi] - g\overline{\psi}\gamma^\mu(x)A_\mu\psi + m\overline{\psi}\psi \} \quad. \tag{5.10}$$

and the wave equations are just the Hamiltonian equations of motion for the momenta.

The non-vanishing components of the connections are $\Gamma^{\cdot}_{\eta\tau\eta} = \Gamma^{\cdot}_{\eta\eta\tau} = -\Gamma^{\cdot}_{\tau\eta\eta} = -\tau$ and $\omega_\eta{}^{30} = -\omega_\eta{}^{03} = 1$. Moreover, we have $\gamma^\tau(x) = \gamma^0$ and $\gamma^\eta(x) = \tau^{-1}\gamma^3$. The explicit form of the Dirac equation in our case is as follows,

$$[i\,\nabla\!\!\!\!/ - m]\psi(x) = [i\gamma^0(\partial_\tau + \frac{1}{2\tau}) + i\gamma^3\frac{1}{\tau}\partial_\eta + i\gamma^r\partial_r - m]\psi(x) = 0 \quad. \tag{5.11}$$

The solutions to this equation will be looked in the form $\psi(x) = [i\,\nabla\!\!\!\!/ + m]\chi(x)$, with the function $\chi(x)$ that obeys the "squared" Dirac equation,

$$[i\,\nabla\!\!\!\!/ + m][i\,\nabla\!\!\!\!/ - m]\chi(x) = \left[\partial_\tau^2 + \frac{1}{\tau}\partial_\tau - \frac{1}{\tau^2}\partial_\eta^2 - \partial_r^2 + m^2 - \gamma^0\gamma^3\frac{1}{\tau^2}\partial_\eta \right]\chi(x) = 0 \quad. \tag{5.12}$$

The spinor part β_σ of the function $\chi(x)$ can be chosen as an eigen-function of the operator $\gamma^0\gamma^3$, viz., $\gamma^0\gamma^3\beta_\sigma = \beta_\sigma$, $\sigma = 1,2$. Therefore, the solution of the original Dirac equation can be written down as $\psi_\sigma^\pm = w_\sigma\chi^\pm(x)$, with the bi-spinor operators $w_\sigma = [i\,\nabla\!\!\!\!/ + m]\beta_\sigma$ that act on the positive- and negative-frequency solutions $\chi^\pm(x)$ of the scalar equation

$$\left[\partial_\tau^2 + \frac{1}{\tau}\partial_\tau - \frac{1}{\tau^2}(\partial_\eta + \frac{1}{2})^2 - \partial_r^2 + m^2 \right]\chi^\pm(x) = 0 \quad. \tag{5.13}$$

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Using the spinor representation of the gamma-matrices, the explicit form of the spinors is found as follows,

$$
w_1(\vec{p}, \hat{l}) = \begin{pmatrix} m \\ 0 \\ i\hat{l} \\ i\partial_+ \end{pmatrix} ; \quad w_2(\vec{p}, \hat{l}) = \begin{pmatrix} -i\partial_- \\ i\hat{l} \\ 0 \\ m \end{pmatrix} , \tag{5.14}
$$

where the operators,

$$
\hat{l} = \partial_\tau + \frac{1}{\tau}(\partial_\eta + \frac{1}{2}) \quad \text{and} \quad \partial_\pm = \partial_x \pm i\partial_y , \tag{5.15}
$$

is convenient to preserve in the differential form. The full set of the one-particle solution can be conveniently written down as follows,

$$
\omega^{(\pm)}_{\sigma,\nu,\vec{p}}(x) = \frac{e^{-\pi\nu/2} w_\sigma(\vec{p}, \nu)}{2^{5/2} \pi m_\perp^{1/2}} e^{\pm i\nu\eta} e^{\pm i\vec{p}\vec{r}} H^{(2)}_{(1)}{}_{\mp i\nu - 1/2}(m_\perp \tau) , \tag{5.16}
$$

The spinors $w_\sigma(\vec{p}, \nu)$ are those of Eq.(5.14) with the replacement of spacial derivatives by corresponding momenta. These solutions are normalized according to the current conservation law. This leads to the scalar product of the following form,

$$
(\psi_1, \psi_2) = \int \tau d\eta d^2\vec{r} \, \overline{\psi}_1(\tau, \eta, \vec{r}) \gamma^\tau \psi_2(\tau, \eta, \vec{r}) . \tag{5.17}
$$

With this definition, the Dirac equation is self-adjoint. The orthonormality relations are read as follows,

$$
(\omega^{(\pm)}_{\sigma,\nu,\vec{p}}, \omega^{(\pm)}_{\sigma',\nu',\vec{p}'}) = \delta_{\sigma\sigma'}\delta(\vec{p} - \vec{p}')\delta(\nu - \nu') , \quad (\omega^{(\pm)}_{\sigma,\nu,\vec{p}}, \omega^{(\mp)}_{\sigma',\nu',\vec{p}'}) = 0 . \tag{5.18}
$$

An equivalent set of the one-particle solutions is obtained by means of the Fourier transform,

$$
\Omega^{(\pm)}_{\sigma,\theta,\vec{p}}(x) = \pm e^{\mp i\pi/4} \int_{-\infty}^{+\infty} \frac{d\theta}{(2\pi)^{1/2}} e^{\mp i\nu\theta} \omega^{(\pm)}_{\sigma,\nu,\vec{p}}(x) . \tag{5.19}
$$

Instead of the boost quantum number ν, these solutions carry the rapidity quantum number θ, which parameterize the energy and the longitudinal momentum of the on-mass-shell fermion as $p^0 = m_\perp \cosh\theta$ and $p^3 = m_\perp \sinh\theta$, respectively. In this way we obtain the positive- and negative-frequency plane wave solutions,

$$
\Omega^{(\pm)}_{\sigma,\theta,\vec{p}} = \frac{w^{(\pm)}_\sigma(\vec{p}, \hat{l}) e^{(\theta-\eta)/2}}{2^{1/2}(2\pi)^{3/2} m_\perp^{1/2}} e^{\mp im_\perp \tau \cosh(\theta-\eta)} e^{\pm i\vec{p}\vec{r}} , \tag{5.20}
$$

which are confined to within the future light wedge of the collision plane $t = z = 0$. They are normalized and orthogonal according to relations,

$$
(\Omega^{(\pm)}_{\sigma,\theta,\vec{p}}, \Omega^{(\pm)}_{\sigma',\theta',\vec{p}'}) = \delta_{\sigma\sigma'}\delta(\vec{p} - \vec{p}')\delta(\theta - \theta') , \quad (\Omega^{(\pm)}_{\sigma,\theta,\vec{p}}, \Omega^{(\mp)}_{\sigma',\theta',\vec{p}'}) = 0 . \tag{5.21}
$$

Action of the operator $\hat{l}$ onto the function $e^{-\eta/2} f(\eta)$ is as follows:

$$
\hat{l} e^{-\eta/2} f(\eta) = e^{-\eta/2}(\partial_\tau + \tau^{-1}\partial_\eta) = e^{\eta/2}(\partial_t + \partial_z) ,
$$

and the spinors preserve their original form up to the change of variables in operator $\hat{l}$. Eventually, the solutions (5.20) can be rewritten in the form,

$$
\Omega^{(\pm)}_{\sigma,\theta,\vec{p}}(x) = \Lambda(-\eta)\psi^{(\pm)}_{\sigma\mathbf{p}}(x) , \tag{5.22}
$$

where

$$
\Lambda(\eta) = \cosh(\eta/2) + \gamma^0\gamma^3 \sinh(\eta/2) = \text{diag}[e^{\eta/2}, e^{-\eta/2}, e^{-\eta/2}, e^{\eta/2}] .
$$

is the matrix of the spinor Lorentz rotation with rapidity η, and $\psi_{\sigma\mathbf{p}}^{(\pm)}$ are the standard plane wave solutions of the Dirac equation normalized, however, on the hypersurfaces $\tau = const$:

$$\psi_{\sigma\mathbf{p}}^{(\pm)}(x) = \frac{u_\sigma^{(\pm)}(\vec{p},\theta)e^{\theta/2}}{2^{1/2}(2\pi)^{3/2}m_\perp^{1/2}}e^{\mp im_\perp\tau\cosh(\theta-\eta)}e^{\pm i\vec{p}\vec{r}} \ , \tag{5.23}$$

$$u_1^{(+)}(\vec{p},\theta) = \begin{pmatrix} m \\ 0 \\ m_\perp e^{-\theta/2} \\ -(p_x+ip_y) \end{pmatrix} \ , \quad u_2^{(+)}(\vec{p},\theta) = \begin{pmatrix} p_x - ip_y \\ m_\perp e^{-\theta/2} \\ 0 \\ m \end{pmatrix} \ ,$$

$$u_1^{(-)}(\vec{p},\theta) = \begin{pmatrix} m \\ 0 \\ -m_\perp e^{-\theta/2} \\ p_x+ip_y \end{pmatrix} \ , \quad u_2^{(-)}(\vec{p},\theta) = \begin{pmatrix} -(p_x-ip_y) \\ -m_\perp e^{-\theta/2} \\ 0 \\ m \end{pmatrix} \ .$$

Either set of the one-particle solutions can be used in order to compute various correlators of the free spinor field. The result reads as follows. The two Wightman correlators,

$$G_{10}(x_1,x_2) = -i\langle 0|\psi(x_1)\overline{\psi}(x_2)|0\rangle = -i\int d\theta d^2\vec{p}\,\Omega_{\theta,\vec{p}}^{(+)}(x_1)\overline{\Omega}_{\theta,\vec{p}}^{(+)}(x_2) \ ,$$

and

$$G_{01}(x_1,x_2) = i\langle 0|\overline{\psi}(x_2)\psi(x_1)|0\rangle = i\int d\theta d^2\vec{p}\,\Omega_{\theta,\vec{p}}^{(-)}(x_1)\overline{\Omega}_{\theta,\vec{p}}^{(-)}(x_2) \ ,$$

have the Fourier representation,

$$G_{\genfrac{}{}{0pt}{}{10}{01}}(x_1,x_2) = \int \frac{d^4p}{(2\pi)^4}e^{-ip(x-x')}[-2\pi i\delta(p^2-m^2)\theta(\pm p^0)]\Lambda(-\eta_1)(\not{p}+m)\Lambda(\eta_2) \ . \tag{5.24}$$

All other correlators and propagators are easily found via these two following the guideline of the previous section.

Presence of the matrices Λ which perform the local Lorentz rotation of the spinors is vital for consistency between the gauge transformations of the spinors and of their gauge field. Local Lorentz rotation (5.22) of the spinors is in one-to-one correspondence with the "rotation" of the gauge condition $A^\tau(x) = 0$. As a result, the matrices $\Lambda(\eta)$ from the fermion correlators can be absorbed into the vertex of interaction. Indeed, for the closed fermion loop, every vertex of interaction between the fermion and the gauge field appears only between the two fermion correlators. In the same way, every fermion line connects two vertices. Therefore, the proof is as follows,

$$\Lambda(\eta)(\gamma^\tau A_\tau + \gamma^\eta A_\eta)\Lambda(-\eta) = \gamma^0\Big(\cosh\eta A_\tau - \frac{\sinh\eta}{\tau}A_\eta\Big) + \gamma^3\Big(-\sinh\eta A_\tau + \frac{\cosh\eta}{\tau}A_\eta\Big) = \gamma^0 A_0 + \gamma^3 A_3 \ , \tag{5.25}$$

and it results in the standard form of the covariant interaction vertex. The latter is affected only by a specific choice of the gauge for the field $A^\mu(x)$. In the composite spinor operators, like fermion self-energy, the matrices Λ of local rotation should be retained explicitly.

VI. SUMMARY

I suggest a new approach to the study the dynamics of the high-energy deeply inelastic processes with the aim to overcome limitations imposed by the parton model. The new theory deals with a single Hilbert space for all initial- and final-state particles. The method is expected to work even when no scale of the hard probe is specified and the standard factorization scheme is inapplicable. The approach is designed exclusively for collisions at extreme energies and requires special selection of events, those where the short-scale structure of the colliding systems is resolved and spectrum of the secondaries has a rapidity plateau. I emphasize the role of a trigger in collection the data and show that special selection of events may significantly affect the type of the available dynamical information.

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